

P.R.GOV.T.COLLEGE (AUTONOMOUS), KAKINADA
II B.Sc. MATHEMATICS-Semester IV (w.e.f. 2017-2018)

Course: Real Analysis

Total Hrs. of Teaching-Learning: 90 @ 6 hr/Week

Total credits: 5

OBJECTIVES:

- Be able to understand and write clear mathematical statements and proofs.
- Be able to apply appropriate method for checking whether the given sequence or series is convergent.
- Be able to develop students ability to think and express themselves in a clear logical way.
- This curriculum gives an opportunity to learn about the derivatives of functions and its applications.

Unit 1: Real Number System and Real Sequence

(18 hours)

The algebraic and order properties of $\mathbb{R}$ – Absolute value and Real line – completeness property of $\mathbb{R}$ – applications of supreme property – intervals -Limit point of a set, Existence of limit points. (No questions to be set from this portion)

Sequences and their limits – Range and Boundedness of sequences - Necessary and sufficient condition for convergence of Monotone sequence, limit point of a sequence, Subsequences and the Bolzano Weierstrass theorem - Cauchy sequences – Cauchy's general principle of convergence theorems.

Unit 2: Infinite Series

(18 hours)

Introduction to Infinite Series – convergence of series –Cauchy's general principle of convergence for series – Tests for convergence of nonnegative terms – p- test – limit comparison test – Cauchy's nth root test - De-Alambert's ratio test - alternating series – Liebnitz's test -absolute and conditional convergence.

Unit 3: Limits and Continuity

(18 hours)

Real valued functions – Boundedness of a function - Limit of a Function, One-sided Limits- Right hand and Left Hand Limits - Limits at Infinity - Infinite Limits.(no question to be set) Continuous Functions- Discontinuity of a Function - Algebra of Continuous Functions – Continuous functions on intervals - Some Properties of Continuity of a function at a point - Uniform Continuity.

Unit 4: Differentiation and Mean Value Theorem

(18 hours)

The Derivability of a function, on an interval, at appoint, Derivability and Continuity of a function - Geometrical meaning of the Derivative - Mean Value Theorems - Rolle's Theorem, Lagrange's Mean Value theorem, Cauchy's Mean Value theorem.

Unit 5: Riemann Integration

(18 hours)

Riemann sums, Upper and Lower Riemann integrals, Riemann integral, Riemann Integrable function – Darboux's Theorem - Necessary and sufficient conditions for Riemann integrability – properties of integrable functions – Fundamental Theorem of Integral Calculus – Integral as the limit of a sum – Mean Value Theorems.

Additional Inputs :

1. problems using cauchy's first theorem on limits and cauchy's second theorem on limits.
2. Statement of Maclaurin's theorem and expansions of $e^x, \sin x, \cos x, \log(1+x)$.

Prescribed book:

- Real Analysis by Rabert & Bartely and D.R.Sherbart, published by John Wiley.

Reference books:

- Elements of Real Analysis by Santhi Nararayan & M.D.Raisinghania, published by S.Chand& Company Pvt. Ltd., New Delhi.
- Course on Real analysis by N.P.Bali-Golden series publications
- A Text Book of Mathematics Semester IV by V.Venkateswarrao & others, published by S.Chand& Company Pvt. Ltd., New Delhi

BLUE PRINT FOR QUESTION PAPER PATTERN

SEMESTER-IV

Unit	TOPIC	V.S.A.Q	S.A.Q	E.Q	Marks allotted
1	Real Number System and Real Sequence	1	1	1	14
2	Infinite Series	1	1	2	22
3	Limits and Continuity	1	1	1	14
4	Differentiation and Mean Value Theorem	1	1	2	22
5	Riemann Integration	1	1	2	22
	TOTAL	5	5	8	94

V.S.A.Q = Very short answer questions (1 mark)

S.A.Q = Short answer questions (5 marks)

E.Q = Essay questions (8 marks)

Very short answer questions : $5 \times 1 = 05$

Short answer questions : $3 \times 5 = 15$

Essay questions : $5 \times 8 = 40$

Total Marks = 60

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P.R. Government College (Autonomous), Kakinada
II year B.Sc., Degree Examinations - IV Semester
Mathematics Course : Real Analysis
Paper-IV (Model Paper w. e. f. 2018-2019)

Time: 2Hrs 30 min

Max. Marks: 60

PART-I

Answer ALL the questions. Each question carries 1 mark.

5 x 1M = 5 M

1. Define convergence of a sequence.
2. Test the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$.
3. Define continuity of a function at a point 'a'.
4. Give an example of a function which is continuous but not derivable.
5. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} [e^{\frac{3}{n}} + e^{\frac{6}{n}} + e^{\frac{9}{n}} + \dots + e^{\frac{3n}{n}}]$

PART-II

Answer any THREE questions each question carries 5 marks.

3 x 5M = 15 M

6. Show that $\lim_{n \rightarrow \infty} \left[\sqrt{\frac{1}{n^2+1}} + \sqrt{\frac{1}{n^2+2}} + \dots + \sqrt{\frac{1}{n^2+n}} \right] = 1$.
7. State and Prove Leibnitz's test.
8. Examine for continuity the function f defined by $f(x) = |x| + |x-1|$ at 0, 1.
9. Show that every derivable function on a closed interval is continuous.
10. State and prove fundamental theorem of Integral Calculus

PART-III

Answer any FIVE questions from the following by choosing at least TWO from each section. Each question carries 8 marks.

5x8M=40M

SECTION -A

11. Prove that every monotonically increasing sequence which is bounded above converges to its least upper bound.
12. State and Prove Cauchy's n^{th} root test for the convergence of series.
13. Examine the convergence of $\sum_{n=1}^{\infty} (\sqrt{n^3+1} - \sqrt{n^3})$.
14. Prove that every continuous function is bounded and attains its bounds.

SECTION-B

15. State and prove Rolle's theorem
16. Using Lgrange's Mean Value Theorem prove that $1+x < e^x < 1+xe^x$, $\forall x > 0$
17. Prove that $f(x) = \sin x$ is integrable on $[0, \frac{\pi}{2}]$ and $\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$
18. State the first mean value theorem of integral calculus and by using it prove that

$$\frac{\pi^3}{24} \leq \int_0^{\pi} \frac{x^2}{5+3 \cos x} \, dx \leq \frac{\pi^3}{6}.$$